Logarithmic Functions

Many occurrences in our natural world can be modeled using logarithmic functions, including the strength of earthquakes, the intensity of sound, or the concentration of hydronium ions in a solution. In this activity you'll explore the relationship between exponential and logarithmic functions, and determine how to write the formula for a logarithmic function that's the inverse of a particular exponential function.

GRAPH INVERSE EXPONENTIAL FUNCTIONS

Logarithms are related to exponents, so start by graphing an exponential function and finding the inverse graph.

1. Open **Logarithmic Functions.gsp.** Press the *Show Exponential Function* button to see the exponential function $y = 2^x$ along with its graph.

With the points selected, choose **Measure | Coordinates** to find their ordered pairs. With the coordinates selected, choose **Graph | Tabulate** to place the coordinates in a table.

2. Press the *Show Points* button to show seven points on the curve. Measure their coordinates, and put the resulting ordered pairs into a table.

Q1 Notice that some of the x-values are negative. Does this mean that the resulting values of the function are negative? Explain why this is true or not true.

Next, interchange the x- and y-values by reflecting the points over the line $y = x$.

3. Press the *Show y=x Line* button. With the line selected, choose **Transform | Mark Mirror.** The line flashes briefly to indicate that it is marked as the mirror.

To show the labels of the selected points, choose **Display | Show Labels.**

4. Press *Show Points* again to select the seven points in order, and choose **Transform | Reflect.** The seven points are reflected. Show their labels.

5. Measure the coordinates of the reflected points and tabulate the results. Align the two tables in order to see the original and reflected points next to each other.

Q2 What do you notice about the coordinates of each pair of points?

Q3 Will any of the x-coordinates of the reflected points be negative? Explain.

Q4 Why is the line $y = x$ called the *axis of symmetry* for a function and its inverse?

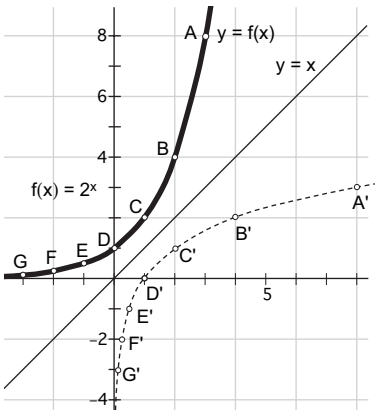
Next reflect the entire graph over the line $y = x$.

Use the **Point** tool to construct the new point.

6. Construct a point on the original graph, and reflect it over the line $y = x$. Drag your new point and observe the behavior of the reflected image.

The reflected graph is the graph of the inverse of the original function.

7. To create the entire reflected graph, select the point on the graph and its reflected image, and choose **Construct | Locus.** Change the color of the locus, and make it dashed.



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You'll use page 2 to graph $y = 10^x$, reflect it to show its inverse, and compare the inverse to the graph of $y = \log x$.

8. On page 2 construct the graph of $y = 10^x$ by choosing **Graph | Plot New Function** and entering 10^x into the Calculator.
9. Construct a point on the graph and reflect it across the graph of $y = x$.

To turn tracing on or off, select the point and choose **Display | Trace Point.**

10. Turn on tracing for the reflected point, and drag the point on the graph to observe the shape of the inverse function.
11. Construct the graph of $y = \log x$.

Q5 What do you observe about the graph of the log function and the reflected image of the exponential graph? What conclusion can you draw?

On page 3 you'll graph the exponential function $f(x) = k^x$ and use different values for k to find a general formula for the logarithmic function that's the inverse of $f(x)$.

Choose **Graph | New Parameter** to create the new parameter, and set its label to *k* and its value to 2 in the dialog box that appears.

12. Create a parameter k, set its value to 2, and use it to construct the graph of $y = k^x$. To enter k into the function definition, click its value in the sketch.
13. Construct a point on the graph, reflect it across $y = x$, and construct the locus.

This locus is the graph of the inverse function. Express this inverse as a logarithmic function by stretching or shrinking the parent logarithmic function $y = \log x$.

14. Using the values of a and b, plot the logarithmic function $y = a\log(x/b)$. Adjust the sliders so that your newly plotted function matches the inverse of $y = k^x$.

Q6 What values of a and b made the graphs match?

To add a row to a table, making the current values permanent, double-click the table.

15. Record the values of k, a, and b in a table. Then change the value of k to 5, match the graphs again, and add a new row of data to the table. Continue adding new data to the table for the following values of k: 10, 100, and 1000.

Q7 What pattern can you find to relate the value of k to the values of a and b?

Q8 Use this pattern to predict the values of a and b needed when $k = 10{,}000$. Test your prediction by gathering another row of data for your table. Then predict the values needed when $k = 0.1$, and test your prediction.

Q9 Use your results to write a formula for the inverse of $f(x) = k^x$.

EXPLORE MORE

Q10 Use algebraic manipulation to explain why your formula from Q9 must be true.

Q11 A general exponential function can be written as $f(x) = a \cdot 10^{(x-h)/b} + k$. Write the corresponding inverse function in terms of a, b, h, and k.

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